Flipping a coin between strat in Urn I.

R	B	ρ	$\alpha = \frac{1}{4}$
10	0	2.5	
0	10	2.5	

H		T	
R_I	B_I	R_I	B_I
10	0	0	10
R_{II}	B_{II}	R_{II}	B_{II}
10	0	0	10

$$\begin{array}{r} 7.5 \\ .25 \\ \hline 375 \\ 150 \\ \hline 1.875 \end{array} \qquad \begin{array}{r} 2.5 \\ .75 \\ \hline .1875 \end{array}$$

$$\begin{array}{r} 1.875 \\ 1.875 \\ \hline 3.750 \end{array}$$

$\frac{1}{2} \cdot p_R$ $\quad \frac{1}{2} \cdot p_B$ $\quad \frac{1}{2} \cdot p_R$ $\quad \frac{1}{2} \cdot p_B$

HR_I $\quad HB_I$ $\quad TR_I$ $\quad TB_I$ $\qquad p_R = 0 \quad p_R = \frac{1}{5} \quad p_R =$

10 $\quad$ 0 $\quad$ 0 $\quad$ 10 $\quad$ 5 $\quad$ 1+0+0 4

$$E(x) = \frac{1}{2} p \cdot 10 + \frac{1}{2}(1-p) \cdot 0 + \frac{1}{2} \cdot p \cdot 0 + \frac{1}{2}(1-p) \cdot 10$$

R_I $\quad E(x) = \frac{1}{4} p \cdot 10 + \frac{1}{4} \cdot (1-p) \cdot 0 + \frac{3}{4} \cdot p \cdot 0 + \frac{3}{4} \cdot (1-p) \cdot 10$

1

$$2.5p + 7.5(1-p) = 7.5 - 5p = .75 \cdot .25 = 3.75$$

$\cdot .25 \quad 7.5$

	H		T	
with	R_I	B_I	R_{II}	B_{II}
	10	0	0	10

The potential distribution of outcomes range from
$(10, 0; \frac{1}{2}, \frac{1}{2})$ to $(10, 0; \frac{1}{2}, \frac{1}{2})$ as p_R ranges from 0 to 1.

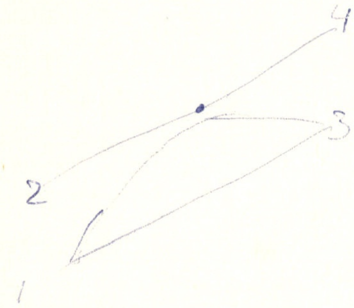
So E is constant; 0

$\frac{1}{2} p_R + \frac{1}{2}(1-p_R) = \frac{1}{2}$
10 0

Compute the possible distribution over the different outcomes.

30		60	
R	Y	B	
10	0	10	$\rightarrow (10, 0; \frac{1}{3}, \frac{2}{3} - 1, 0)$
0	10	10	$\rightarrow (10, 0; \frac{2}{3}, \frac{1}{3})$

Flipping coin between I and IV results in action whose expected payoffs is midway between I & IV; that action has a single dist over outcomes, and so does any risk-combination.



But the dist. of outcomes to

II is $(10, 0; \underline{0} - \frac{2}{3}, 1 - \frac{1}{3})$

To III: $(10, 0; \underline{\frac{1}{3}} - 1, \frac{2}{3} - 0)$

Dist over ~~sts~~ outcomes for: (II, III; ½, ½) is $(10, 0; \underline{\frac{5}{12}} - \frac{9}{12}, \frac{7}{12} - \frac{3}{12})$.

Thus, minimum prob. of 10 is higher than for either II or III (max prob. of 10 is lower). To extent that min is weighted more than max, combination will be preferred to either II or III.

$$\begin{array}{c|ccc|ccc|c}
 & \frac{1}{3}\ p & H\ p_2 & (1-p_1-p_2) & 4 & T & & \\
\hline
\mathrm{I} & 10 & 0 & 0 & 0 & 10 & 10 & (10, 0; \tfrac{1}{2}, \tfrac{1}{2}) \\
\mathrm{II} & 0 & 10 & 0 & 10 & 0 & 10 & (10, 0; \tfrac{1}{2}, \tfrac{1}{2})
\end{array}$$

I ~~should~~ dominates II, but I : II

$\frac{1}{2}\, p_1 \cdot 10 \quad \frac{1}{2} p_2 \cdot 0 \qquad p \qquad \frac{1}{2} p_2 \cdot 10 \quad \frac{1}{2}(1-p_1-p_2)10$

$\frac{3}{4}$

$$\begin{array}{cccc}
 & \frac{1}{3} & & \\
\frac{1}{4} & 10 & 0 & 0 \\
\frac{3}{4} & 0 & 10 & 10
\end{array} \qquad \frac{1}{12}\cdot 10 + \frac{6}{12}\cdot 10 = \frac{7}{12}\cdot 10$$

I $(10, 0; \frac{3}{4}, \frac{1}{4} - \frac{1}{4}, \frac{3}{4})$ 2.5

$(0 - \frac{2}{3}) \cdot 10$

$(\frac{1}{3} - 1) \cdot 10$

II $(10, 0; \frac{3}{4}$

$$\begin{array}{cccc}
 & \frac{1}{3} & \frac{2}{3} & \\
\frac{1}{4} & 0 & 10 & 0 \\
\frac{3}{4} & 10 & 0 & 10
\end{array} \qquad \begin{array}{l} \frac{1}{4}\cdot 10 + \frac{1}{6}\cdot 10 \leftrightarrow \frac{1}{2}\cdot 10 \\ \frac{5}{12}\cdot 10 - \frac{9}{12}\cdot 10 \end{array}$$

The range of distributions over distinct outcomes is not the same for all actions; some have only one dist (risk)

Don't stress that "expectation is known" or "guaranteed" ~~but~~ — as though uncertainty had been converted into ~~risk~~ certainty;

but that prob. dist. of outcomes is known; certain outcomes are felt confidently to be "much more likely" than others, or "equally likely"; in a large number of cases when you attached these likelihoods, when you had this same info and confidence (e.g. in repetition of same bet, if possible) you would expect to win about half the time; and would so bet on your compound winnings.

This is not true of "equal ignorance."
(imagine many urns of unknown contents).

Thus, in games, mixed minimax strategy often converts ignorance into risk (if all opponent's pure strats are included in his optimum strat.

But note arguments about "best optimum" strat; why not give up some on ~~worst~~ min. to gain on others

Can't assign vN-M utilities to strats with ambiguous outcomes; if they are prizes, they don't obey vN-M axioms.

When we are ignorant — don't feel sure as to relative likelihoods — it makes sense to be influenced by the range of possible outcomes more than by estimates of likelihood. This could amount to "taking all outcomes into account equally": equalizing probs.

But this rule would not tend to lead one's agent away from situations of ignorance when he had alternatives. You don't want him to do this "blindly" — e.g. when worst outcomes compare favorably with best or expected outcomes under "risk" situation; or when "expected" outcome in ambiguous situation is much better and worst isn't much worse. Weighting worst dists more, then, prejudices choice against ignorant gambles where expectation isn't much better and where worst "reasonable case" is much worse than best guess.

Why don't regrets work?

Does random sample, like mixed strat, convert uncertainty into risk?

An action involving an observation may offer low max regret, or constant small regret (amounting to cost of delay + observation).
What else can be said about it?
adaptiveness; speed + effectiveness of learning.
Flexibility; sequential decision; postponement of choice; liquidity; centralisation; speed of reaction, communication, decision.
What do these have in common? What advantages; under what conditions?
Do they lead to actions with similar payoffs, or regrets?
(Hyp: all look good under ignorance — not risk — if they don't cost too much).

Speed of adjustment, etc., presumably related to likelihood of changes in environment that are unpredictable long in advance; or, knowledge of whose long-run stochastic properties is not enough for adequate adaptation.

over

In some cases, knowledge of opponent's long-run mixed strategy might allow optimal or adequate long-run adjustment. Question is, value of short-run "intelligence" on his precise choice in an individual play, if combined with ability to adjust fast?

(Value of bomb alarm info; depends on ability to adjust, and on the potential value of adjustment).

Note that you don't automatically "know" what is happening, or what has happened, unless you take steps to acquire this info; especially in war.

Clausewitz

It can be too costly to build in delays for decision and adjusted response, when info is unlikely to arrive "in time" despite effort, and where value of adjustment is small (assumption of spasm war proponents?).

Spasm war approach does reflect some practical wisdom, combined with bad assumptions; they tend to ask sensible questions, but they don't look for answers, and assume bad ones.

TWA

Sunday	#100	9:00AM	LA	NY - 4:50
	#800	7:30 PM	Paris	7:30AM

Man who obeys axioms all the time and whose vN-M utility is concave will accept unfavorable money bets; if these predominate among the bets offered him "in the long run" he will lose money. (i.e. have less than if he had never gambled.)

Long-run arguments really favor maximizing expectation of money (with max of expected convex utility being somewhat more conservative; means you will take bets only at good money odds. Your long-run expectation will be lower but the prob of ruin is also lower).

On committee problem:

Disagreement over prob. dist. of events "matters" only insofar as it affects prob. dist. over outcomes; ~~a)~~ a) this depends on payoffs; b) conflict will matter for some actions, not for others; c) one conflict will affect one pair of actions "a lot", another pair only a "little" (or not at all); another similar conflict, other pairs.

e.g.

	A	B	C	D
I	a	a	b	b
II	b	b	a	a

Conflict of own prob. dist may range from (1, 0, 0, 0) to (0, 0, 0, 1):
but the only "intelligence" question that matters is:

$$pr(A \vee B) \overset{?}{\gtrless} pr(C \vee D)$$

If there are actions similar to I in which

	A	B	C	D
I′	a + ε	a	b	b
I″	a	a + ε	b	b

where ε is small relative to (a − b)?

there may be "secondary" interest in question: $pr\ A \overset{?}{\gtrless} pr\ B$

The "difference it ^may makes" ~~which prob. dist. is selected out of~~ to improve intelligence info is measured by: a) Minimum difference between the expected values of actions in the indeterminate "conflict set" as prob. dist. ranges over possible set; perhaps, minimum maximum dist., looking at differences between all pairs of actions for each dist.

(b) Maximum maximum dist.

In example above, (b) is $(a-b)$ (for prob. dists $(\lambda, 1-\lambda, 0, 0)$ and $(0, 0, \lambda, 1-\lambda)$.

(a) is ~ 0, for prob. dists in which $(P_1 + P_2) - (P_3 + P_4) \to 0$

(c) "Average difference" corresponding to prob. dist. chosen "at random" among dists in conflict. (To compute, have to compute difference for all dists?).

(b) may be taken as the "value" of the info: pr $(A \cup B) >$ pr $(C \cup D)$; if otherwise II had been taken, the difference in expected value for some members of committee would ~~range from a~~ to be $(a-b)$.

Committee rule: Investigate voting procedures, where members disagree on ordering of actions.

(Agreement, or group decision, on distribution isn't necessary; neither dist. over events or even dist. § over outcomes need be uniquely determined to determine choice of action (Thus, deadlock can often be broken by inventing new action which is agreed by all to be optimal — not necessarily dominant — despite disagreement over dists. Dominant action is important only where a) disagreement over dists covers whole range of possible dists, or (b) analyst is ignorant about or makes no assumptions about range of conflict over dists.

Generally useless or costly to look for dominant action; look for optimal action w.r.t. § specific range of dists considered. (contrary to Hitch, Wohlstetter).

Potential value of intelligence depends on possible difference in expected value of action that *might be chosen* after receipt of possible intell., and the expected value of the action X that *would* have been chosen *without* new info; both of these depend on *committee's decision rule*.

Compute this value for "amount of intell. that could *change* committee choice from X to (Y, Z, ...); and for intell that could change it to (Y, Z ...; A, B ....). Then estimate probs that a given act of intelligence gathering would generate "this much" info.

i.e. roughly classify "lots of intelligence" in terms of ability to *change* committee action from ~~1 action to another~~ the action that would otherwise result from committee decision rule to ~~some other~~ given sets of other actions. Then compute Value of intelligence of a given lot.

Knight p. 253 Risks involved in evaluating quality of an individual's judgment especially subject to ignorance. Result of applying Ellsberg rule conservatively is that it is hard to get anyone to gamble large sums in supplying an individual with money (many individuals are really "good bets" — but which ones?).

Yet even a man with good judgment needs large sums, large scope of decisions, to avoid effects of "bad luck". To get it: a) personal fortunes help; b) partnership allows ~~two~~ men who trust themselves and each other to pool their own capital and individual sources of credit; c) when big money is involved, can't find enough partners one trusts enough to accept full liability for their decisions; so limited liability corporation is formed.

(Is there any disadvantage, to investor or creditor, in corp. relative to partnership?)

(Is there more trust, safety, from point of view of backer, in the decision-making processes of a committee, or corp, than individual?

On the other hand, big money may come only to good, bold individuals — not committees — so premium on inside knowledge & study of management).

"Consolidation of risks" is necessary both with ~~obj~~ "objective" risks and "subjective."
Difference (which answers questions) is that the reduction of risk achieved is "objective" + "measurable" in the first case, much less so in the second; so problems of getting conservative backers for latter consolidation.

Speculator in "non-objective risks" trusts his own judgment; "objectively" (in eyes of observer) he is reducing risks somewhat by his consolidation of cases (assuming considerable randomness in events, unlikelihood of strong negative correlation in his bad judgment with true probs) but perhaps not a lot, to beholder.
(unlike life insurance company).

In risking large sum, person will want to know: "How often will I have the opportunity to make gambles of this size?"

Specialization of risk-bearing also takes place when risks are "objective"; but the result of the consolidation is then not risky, but "objective certainty."
Difference is in the sort of specialists who undertake to consolidate (w.r.t. trust in their judgment, attitude to risk-bearing) and in their ability to get capital.

10	0	0
0	10	0

	α	
I	10	0
II	0	10
	9	9

If <u>maxima are equal</u>, and "observation" has fixed cost,
(necessary?)

minimax regret rule is equivalent to <u>minimax</u> rule, and gives <u>constant</u> outcome (unambiguous).

If α is ambiguous, might <u>pay more for observation</u> (if only <u>one</u> of I, II were "available") than if α is <u>any</u> known prob (e.g. $\alpha = \frac{1}{2}$).

i.e.

I	10	0
III	3	3

whereas if $\alpha = \frac{1}{2}$,

I	10	0
IV	5	5

(By the way: Does Raiffa <u>prefer</u> to flip a coin than to choose I? HA!)

$$\begin{array}{llll} & R & & Y \\ & 30 & & 0-60 \qquad\qquad 30 \quad 0-60 \\ \hline I & \overbrace{10 \qquad\qquad 0}^{6} & & \\ II & \underbrace{0 \qquad\qquad 10}_{2} & & \end{array}$$

$6 > 2$

1	1	1	1	1	1	1	1
0	10	0	.9	2	1	1.8	0
10	0	0	2	.9	1	0	1.1

$p_1 \cdot .1 > 1 \cdot p_2$

$p_1 > 10 p_2$

$p_2 >$

6	6	$\cancel{-10}4$
0	10	−100

$6p_1 > 4p_2 + 4p_3$

$p_1 > \frac{2}{3}(p_2 + p_3)$

0	1	0	0	4	4
.1	0	0	6	0	0

4	1	−2
4	−2	−1
3	0	0
1	1	1

$p_1 \geq \frac{1}{2}(p_2 + p_3) =$

$2p_1 = p_2 \cdot 1 + p_3 \cdot 1$

$p_3 \geq 1.5(p_1 + p_2)$

1	1	1	1	1	1
−1	2	2	3	0	0
2	−1	2	0	3	0
2	2	−1	0	0	3
.5	.5	2	1.5	1.5	0

	5	5	~~-105~~ -100			5	5	5
I	0	10	-100			10	0	0
II	10	0	-100		10 -- -100	0	10	0
III	9	9	-101		9 -- -101	1	1	1
IV	2	2	-108					

-30

-30

~~33~~ $27\frac{1}{3}$

$p_1 \cdot 5 > p_2 \cdot 5 + p_3 \cdot 5 \Leftrightarrow p_1 > 5(p_2 + p_3)$

$p_2 > p_1$

$p_1 \cdot 10 \sim p_2$ $\qquad p_1 \cdot 9 > \frac{p_2 \cdot 1 + p_3 \cdot 1}{9} =$

Condition for III to be preferable to I: $p_1 > \frac{1}{9}(p_2 + p_3)$

preferable to II: $p_2 > \frac{1}{9}(p_2 + p_3)$

Condition for IV to be preferable to II:

~~$p_1 \cdot 2 > p_2 \cdot 8 + p_3 \cdot 8 \Leftrightarrow p_1 > 8(p_2 + p_3)$~~

Rule like $\rho Y_{est} + 1-\rho\left[\alpha Y_{max} + (1-\alpha) Y_{min}\right]$

implies: apply several decision criteria; if they conflict, weight their results as desired.

["Which has higher max? Higher min? Higher average? ~~Higher~~ Lower ~~average~~ max regret? Higher estimated expectation?

Hyp: ~~Max~~ Minimax regret action is preferable if uncertainty is great enough (i.e. if for every P_i, $i = 1 \ldots n$, $P_i > \frac{1}{k}$, $k > n$.